Economical Scheme for Estimating Orbital Lifetimes

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Theme

A CHART has been prepared that permits rapid estimation of the orbital lifetime of satellites when the lifetime is in excess of about three times the solar cycle. This chart differs from earlier methods for these estimations in that it takes into account the time-varying density of the upper atmosphere that results from the 11-year solar cycle. As a result, the predicted orbital lifetimes are three or four times larger than those predicted from previous charts. This fact is quite important in safety evaluations of the re-entry of nuclear heat sources from satellites.

Contents

The application of radioisotopes to provide electric power to spacecraft by means of thermoelectric conversion has led to a requirement to conduct preflight safety evaluations relative to the eventual re-entry of the nuclear heat source under both normal and abnormal flight conditions. Abnormal conditions may be caused by a failure of the launch vehicle that results in the satellite being inadvertently injected into an undesired orbit around the earth. Nuclear heat sources are designed to re-enter intact subsequent to such a launch failure. For this contingency, it becomes necessary to ascertain the lifetime of the orbit so that an assessment can be made of the risk associated with the re-entry.

The primary nonconservative force affecting the orbital lifetime of a satellite is atmospheric drag, which is directly proportional to atmospheric density. Previous lifetime estimates were based on either "lifetime" charts[1] or computer programs[2] using a direct integration of the equations of motion. The charts had been based on a Standard Atmosphere and did not account for the large variation in upper atmospheric properties resulting from solar activity. The computer programs, at the other extreme, are quite sophisticated and costly since they have the capability of accounting for daily and even hourly variations in atmospheric properties.

The major factor that affects high-altitude atmospheric density is the variation caused by the 11-year cycle of solar activity. It was decided to develop a simple and efficient semianalytic procedure directed toward long life time estimates ($\gtrsim$ 100 yrs), accepting some inaccuracies in the solution of the equation of motion as being compatible with the degree of uncertainty in solar activity predictions.

The numerical procedure for solving Lagrange's planetary equation is based on the development presented by King-Hele,[3] the major assumptions being: a) air drag is the only nonconservative force considered; b) during one revolution, the action of air drag changes the orbit by a small amount; and c) air density varies exponentially with height above perigee. The atmospheric model relating density to altitude is based on the method outlined by Jacchia.[4] The major input to Jacchia's atmospheric model is the exospheric temperature, T_∞. Using more than 100,000 measurements of atmospheric density at high altitudes via observation of satellite decay rates, Jacchia developed empirical correlations of exospheric temperature with such factors as solar activity, seasonal variations, and geomagnetic activity. The Jacchia correlations were reviewed in light of the long-lifetime requirements of this study and were accordingly simplified by employing average values for the short time (i.e., seasonal) factors. A simplified relationship resulted relating T_∞ to the solar activity parameter, the $\bar{F}_{10.7\,\mathrm{cm}}$ flux

$$T_\infty = 492 + 3.73\ \bar{F}_{10.7},\ \mathrm{K}$$

Unfortunately, a data base for the $F_{10.7\,\mathrm{cm}}$ solar flux exists back to only 1947. However, historical records have provided estimated values of R_Z (the Zurich sunspot number) back to 1750, encompassing approximately 20 solar cycles.[5] Comparison of current values of R_Z and $F_{10.7\,\mathrm{cm}}$ established the simplified correlation $F_{10.7} = R_Z + 57$, which provided a means of using the historical solar cycle data in the current study.

This procedure was programed in PL/I for the IBM 360/91 computer. To verify the accuracy of the computer code, check cases were run for a series of orbital lifetimes without the solar effect, using tabular values of the 1962 Standard Atmosphere, for which lifetime solutions are available based on a complete numerical solution.[2] The resulting comparison showed excellent agreement, the maximum difference being about 8%. A mean solar cycle model based on the averages of cycles 8-19 was used in the calculation. The results led to the following conclusions, the first three of which can be seen in Fig. 1:

1) With the mean solar cycle as a basis, lifetimes are three to four times longer than would be predicted using the 1962 Standard Atmosphere.

2) A 1σ deviation of the solar cycle from the mean value can affect the lifetime by a factor of about 1.5.

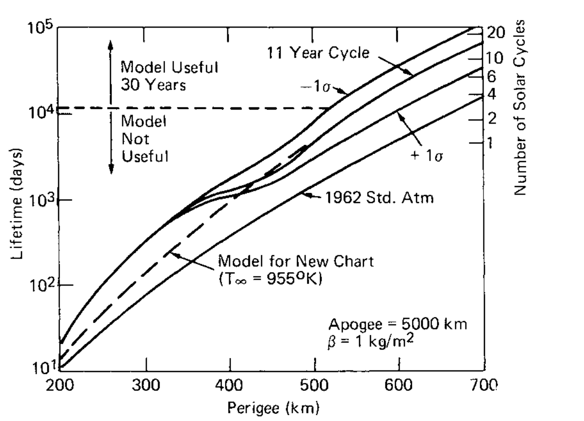

Fig. 1 Lifetime estimates based on solar cycle compared to estimates based on the 1962 Standard Atmosphere.

Synoptic received November 20, 1974; revision received January 30, 1975. Full paper available from the National Technical Information Service, Springfield, Va., 22151, as N75-13006 at the standard price (available upon request). The investigation was supported by the U.S. Atomic Energy Commission, Safety Branch, Space Nuclear Safety Division under problem statement AEC/SNS 3060.

Index category: Earth-Orbital Trajectories.

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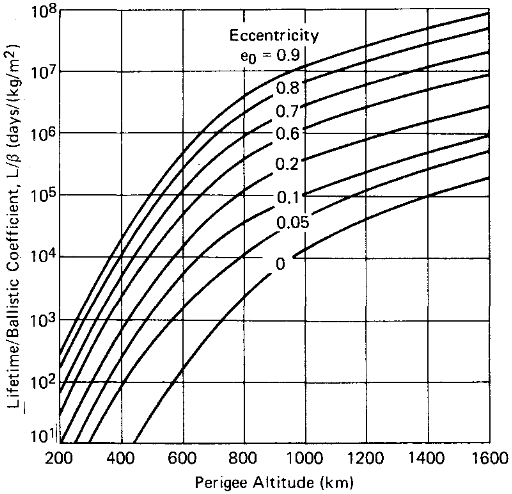

Fig. 2 Orbital lifetime map for long lifetimes (≥30 yr) based on the 11-yr mean solar cycle (T_∞ = 955°K).

3) For lifetime L greater than 30 yr, estimates based on a nontime-varying, reference value exospheric temperature derived from a value of reference density that is time-averaged over the mean solar cycle (dashed curve in Fig. 2) agree well with the more exact calculations that directly considered the mean time-varying solar activy (i.e., the dashed curve blends into the solid 11-year cycle curve). This conclusion permitted the economic development of an orbital lifetime chart, since small values of the ballistic coefficient, β, can be used to determine the ratio L/β that is applicable to the mean solar cycle for $L \gtrsim$ 30 years. The resulting lifetime chart for values of initial eccentricities of $e_0 = 0$ to 0.90 and for initial perigee altitudes from 200 to 1600 km is shown in Fig. 2.

4) For long lifetimes ($\gtrsim$ 30 yr), the lifetime estimate is independent of where on the solar cycle the calculations are begun.

5) For long lifetimes ($\gtrsim$ 30 yr), the lifetime is directly proportional to the ballistic coefficient; $\beta = W/C_DA$.

6) Lifetime estimates based on a mean solar cycle are, in general, significantly higher than those currently used; comparisons with results in Ref. 1 show factors of up to 60 in some regions.

Although the objective of this work has been to provide more realistic (rapid) estimates of orbital lifetimes by accounting for solar effects, no comparisons with experimental data can yet be made because of the long lifetimes under consideration. The major uncertainty is the solar cycle model, for which it has been assumed implicitly that the mean of future cycles will be represented by the mean value from past data used herein.

References

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